

Non Cooperative Game

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Abstract—*In the modern economy, where there is a strong correlation between business undertakings and profit, the profit of a subject depends not only on its behaviour, but also on the behaviour of other participants in the decision making process. Therefore, the decision maker should analyze the strategies that have been chosen or will be chosen by his opponents, but also to perform analysis of the strategies that other decision makers will choose in a response to the strategy which has yet to be chosen. To select an optimal strategy, on the oligopolistic market, decision makers can use game theory. Game theory is a mathematical theory that is used for analysis and solving of conflict situations, in which participants have opposing interests. The concepts of game theory provide a tool for formulating, analyzing and understanding different strategies. It attempts to address the functional relationship between the selected strategies of individual players and their market outcome, which may be either profit or loss. In this paper the key aspects of game theory have been used to show how it can be implemented for understanding the development and functioning of the oligopoly market and how managers need to think about the strategic decisions. The key to game theory and to understanding why better people may make the world a worse place is to understand the delicate balance of equilibrium. It is true that if we simply become more caring and nothing else happens the world will at least be no worse. However: if we become more caring we will wish to change how we behave. As this example shows, when we both try to do this at the same time, the end result may make us all worse off.*

Keywords: *Game, pay off, dominant strategy, Nash Equilibrium, firms' behaviour*

1. BACKGROUND

The Americans were willing to play Doomsday – or at least Kennedy made his own Joint Chiefs of Staff believe that. On the evening of his fateful decision, Kennedy told his Joint Chiefs that he hoped they would all be alive tomorrow. With a convincing enough show that the Americans were willing to go Doomsday, the Soviets changed course in midstream, and backed down. The Soviets obviously changed their expectations. Kennedy's "psyche-out" won, but led us to the brink of nuclear disaster. The "players" in this Cuban missile crisis met in Havana in 1992 for a "30th Anniversary." The consensus from both sides was, "Even we didn't imagine how close we really were."

Yet even now, there are still current books on the history of economic thought that fail to allocate even one full page to

Nash's work (Niehans, 1990), and prominent scholars can search for a "consilient" unification of social science with virtually no regard for the real unification that has been provided by non cooperative game theory (Wilson, 1998). All situations in which at least one agent can only act to maximize his utility through anticipating (either consciously, or just implicitly in his behaviour) the responses to his actions by one or more other agents is called a game. Weintraub (1992) offers a good overview of the early history of game theory, with a particular focus on the work of von Neumann and Morgenstern (Morgenstern, 1976). Different equilibria can appear. Maybe an equilibrium can be reached (Which is why we all drive on the same side of the road within a country). Maybe this equilibrium will be worse for all players (Which is why people litter or pollute common resources), or maybe everyone will try to be as unpredictable as possible in their actions (as might happen with troop deployment in war).

2. STATEMENT OF PROBLEM

One of the forms of the market structure is oligopoly. It is a market in which only a few companies compete, with products that can be differentiated but are not necessarily so. Another important feature of oligopoly markets is that there are barriers to entry for new companies. The monopoly power and profitability of the oligopoly companies depend in part on the interaction between the companies. If they cooperate, companies can charge prices well above marginal cost and, thus, can earn large profits. If they compete aggressively, this will result in low prices and, consequently, in lower profits. Why do companies in some oligopolistic markets collaborate and in others compete fiercely? How do companies set prices? The answers to these questions will be obtained through the application of game theory, which has made a significant contribution to the analysis of strategic decision making in oligopoly companies.

3. THEORETICAL DEVELOPMENTS OF GAME THEORY

Since 1994, when the Nobel Memorial Prize in Economic Sciences was awarded to John Nash, John Harsanyi, and Reinhard Selten, there have been a number of essays in appreciation of Nash's work; Leonard (1994), Kuhn (1994),

Milnor (1995), Rubinstein (1995), van Damme and Weibull (1995), Myerson (1996), and Binmore's introduction to the collected game-theory papers of Nash (1996). Although some developments occurred before it, the field of game theory came into being with the 1944 book "Theory of Games and Economic Behavior" by John von Neumann and Oskar Morgenstern. A detailed biography of John Nash has been written by Sylvia Nasar (1998). In the century following Cournot (1838) (in what Niehans, 1990, calls the Marginalist Era), economic theorists worked to develop a deeper theory of the determinants of supply and demand in markets, based on models of rational competitive decision-making by producers and consumers Nash (1950b) formally defined an equilibrium of a non cooperative game to be a profile of strategies, one for each player in the game, such that each player's strategy maximizes his expected utility payoff against the given strategies of the other players. John von Neumann's (1928) first great paper on game theory begins with a section entitled "General Simplifications" that lays out a full development of this idea.

4. OBJECTIVES

The present paper aims to fulfil following objectives:

- (i) Introduce the concept of game theory
- (ii) Illustrate the methods of solving non cooperative game
- (iii) Illustrate the application of Non Cooperative in economics to the real world

5. METHODOLOGY

Our research is a fundamental one aiming to construct an internal history of the literature of game theory existence documented based on the view of (Nash (1950, 1951). In this respect our goal was to identify the main authors that have contributed significantly to documenting the existence and magnitude of game. Our literature review is a thematic one, the studies and authors that formed the sampling of the literature were selected only with the scope of assessing our research questions. The methodology adopted was deductive and the conclusions were drawn based on the literature. In order to identify all relevant literature, the literature search included the following steps:

1. Keyword search using Business Source Complete database,
2. Identification of publications citing the key publications under a thematic view.

6. LIMITATIONS

- i. The research paper is based on deductive logic of economic concepts.
- ii. No attempt has been made to include empirical evidences.

7. THE CONCEPT OF GAME

Of course, the average player has a view of what the average player's view is, a view of what the average player's view of the average player's view is, etc., etc., *ad infinitum*. Keynes said that there were a few people in the stock market able to play on the fifth or sixth level of such a series of expectations. But not many could, and no one could play higher than that.

Harsanyi gave us a "fixed point" for analysis; a point at which expectations about other expectations are consistent with one's optimizing strategy. Such consistency is not the same as being correct.

Strategic interdependence of perfectly competitive firms or a monopoly firm is either minor or nonexistent. Models of perfect competition and monopoly do not require incorporating game theory. In contrast, strategic interdependence is a major characteristic of imperfect competition. Game theory has become the foundation of models addressing imperfect-competition firm behaviour. Economic models based on game theory are abstractions from strategic interaction of agents. It allows tractable interactions, yielding implications and conclusions that can then be used for understanding actual strategic interactions.

Game theory considers situations where agents (households or firms) make decisions as strategic reactions to other agents' actions (live variables) instead of as reactions to exogenous prices (dead variables). One of the most general problems in economics is outguessing a rival, for example, a firm seeks to determine its rival's most profitable counterstrategy to its own current policy formulates an appropriate defensive measure, for example, in 1996 Pepsi supplied its cola aboard. Russia's space station Mir Coca-Cola countered by offering its cola aboard shuttle Endeavour. Game theory provides an avenue for economists to investigate and develop descriptions of strategic interaction of agents.

7.1 Strategic Interdependence Implies

- Each agent's welfare depends not only on her own actions but also on actions of other agents (**players**).
- Best actions for her may depend on what she expects other agents to do.

Theory emphasizes study of rational decision-making based on assumption that agents attempt to maximize utility. Alternatively, agents' behaviour could be expanded by considering a sociological, psychological, or biological perspective. Recent progress in game theory has resulted in ability to view economic behaviour as a special case of game theory. In economics, this strategic interdependence among agents is called **non-cooperative game theory**. For example, interaction of two football teams playing a game is non-cooperative.

A game-theory model is composed of

- Players
- Rules by which game is played
- Rules involve what, when, and how game is played
- What information each player knows before she moves (chooses some action)
- When a player moves relative to other players
- How players can move (their set of choices)
- Outcome
 - Payoffs
 - Some reward or consequence of playing game
- May be in form of a change in (marginal) utility, revenue, profit, or some nonmonetary change in satisfaction
- Assumed that payoffs can at least be ranked ordinally in terms of each player's preferences

A player's strategy is his complete contingent plan. If it could be written down, any other agent could follow the plan and duplicate player's actions. Thus, a strategy is a player's course of action involving a set of actions (moves) dependent on actions of other players.

8. METHODS OF SOLVING GAME

The natural starting point in a search for a solution concept is standard decision theory: we assume that each player has some probability beliefs about the strategies that the other player might choose and that each player chooses the strategy that maximizes his expected payoff. A natural consistency requirement is that each player's belief about the other player's choices coincides with the actual choices the other player intends to make. Expectations that are consistent with actual frequencies are sometimes called rational expectations. Nash equilibrium is a certain kind of rational expectations equilibrium. Nash equilibrium is a minimal consistency requirement to put on a pair of strategies:

Suppose for example that the payoff to Row is $u_r(r; c)$ if Row plays r and Column plays c . If Row believes that Column will play c^* , then Row's best reply is r^* and similarly for Column. No player would find it in his or her interest to deviate unilaterally from a Nash equilibrium strategy. Nash equilibrium may be in pure strategy or in mixed strategy.

8.1 Nash equilibrium in Pure Strategies.

A Nash equilibrium in pure strategies is a pair (r^*, c^*) such that $u_r(r^*, c^*) \geq u_r(r, c^*)$ for all Row strategies r , and $u_c(r^*, c^*) \geq u_c(r^*, c)$ for all Column strategies c . In the pure strategy, probability of selecting a particular strategy by a player is equal to one.

8.2 Nash Equilibrium in Mixed Strategies.

A Nash equilibrium in mixed strategies consists of probability beliefs (π_r, π_c) over strategies, and probability of choosing strategies $(p_r; p_c)$, such that:

1. The beliefs are correct: $p_r = \pi_r$ and $p_c = \pi_c$ for all r and c ; and,
2. Each player is choosing (p_r) and (p_c) so as to maximize his expected utility given his beliefs.

Row's probability that a particular outcome $(r; c)$ will occur is $(p_r \pi_c)$. This is simply the (objective) probability that Row plays r times Row's (subjective) probability that Column plays c . Hence, Row's objective is to choose a probability distribution (p_r) that maximizes

$$\text{Row's expected payoff} = \sum_r \sum_c p_r \pi_c u_r(r, c).$$

Column, on the other hand, wishes to maximize

$$\text{Column's expected payoff} = \sum_c \sum_r p_c \pi_r u_c(r, c).$$

8.3 Dominant strategies

Let r_1 and r_2 be two of Row's strategies. We say that r_1 strictly dominates r_2 for Row if the payoff from strategy r_1 is strictly larger than the payoff for r_2 no matter what choice Column makes. The strategy r_1 weakly dominates r_2 if the payoff from r_1 is at least as large for all choices Column might make and strictly larger for some choice. A dominant strategy equilibrium is a choice of strategies by each player such that each strategy (weakly) dominates every other strategy available to that player. Clearly, dominant strategy equilibrium is a Nash equilibrium, but not all Nash equilibria are dominant strategy equilibrium.

9. ILLUSTRATIONS OF GAME

9.1 Chicken Game

A famous game is called "Chicken," named after a famous adolescent hot-rod ceremony from the United States of the 1950 say that Boeing and Airbus are both considering entering the jumbo jet market, but that because of increasing returns to scale and relatively low demand, there is only enough room for one of them. The game matrix (called the "normal form" of a game) could look like this. (This example is taken from an article by Paul R. Krugman, "Is Free Trade Passe?" in the *Journal of Economic Perspectives*, Fall 1987.)

Table-1: Chicken Game
Player B

		Left		Right		
		Air Bus	Boeing	Build	Don't Build	
Player A	Top	Air Bus	Boeing	Build	Don't Build	
	Bottom	Build	-10	-10	10	-1
		Don't Build	-1	10	5	5

As shown in Table-1, if player A chooses to play Top, player B will follow Right. As B chooses Right, A's optimal choice will be Top. So Top Right will be Nash Equilibrium. On the other hand, if player A chooses to play Bottom, player B will follow Left. As B chooses Left, A's optimal choice will be Bottom. Similarly, if player B chooses to play Left, player A will follow Bottom. As A chooses Bottom, B's optimal choice will be Left. So Bottom Left will be Nash Equilibrium. Again, A chooses Bottom, B's optimal choice will be Left. On the other hand, if player A chooses to play Bottom, player B will follow Left. As B chooses Left, A's optimal choice will be Bottom. So Bottom Left will be Nash Equilibrium. Therefore, Top-Right and Bottom-Left squares are both Nash Equilibria, sometimes called Non-cooperative Equilibria. It can be easily confirmed that neither player will have an interest in moving if it finds itself in either of these cells; doing so would only make it worse off.

9.2 Battle of Sexes

Two players are a wife and husband deciding what to do on a Saturday night. Each has two choices: going to opera or to the fights.

Table-2: Battle of Sexes

		Husband	
		Opera	Fights
Wife	Opera	(5,2)	(1,1)
	Fights	(-7,-1)	(2,5)

The game is represented as following payoff matrix: As shown in Table-1, if player wife chooses to go for Opera, Husband also will follow to go for Opera. On the other hand, if Husband chooses Opera, wife will follow the same. So Opera will be Nash Equilibrium As wife chooses to fight, Husband's optimal choice will be to choose fight. So fight will be also Nash Equilibrium.

9.3 Prisoner's Dilemma

Consider another famous example so-called "Prisoner's Dilemma." Convicts A and Convict B have just been nabbed for a crime. They have promised each other not to rat on the other if caught. The court offers them the following prison-expectations (in years) if they confess or deny to the other.

Table-3: Prisoner's Dilemma

		Convict B	
		Confess	Deny
Convict A	Confess	(2,2)	(10,0)
	Deny	(0,10)	(7,7)

As shown in Table-3, if A chooses to confess, player B will deny. As B chooses deny, A's optimal choice will be also to deny. On the other hand, if B chooses to confess, player A will follow deny. As A chooses deny, B's optimal choice will be also deny. So deny will be Nash Equilibrium. Thus both will be sentenced for 7 years. The Nash equilibrium will be unique. However, Pareto social equilibrium is 2 years sentence to each, which is not possible due to the attitude of not relying on the rival.

9.4 Behaviour of Firm

Firms' behaviour in economics can be analyzed with the help of Game Theory. Some illustrations of behaviour of firm behaviour are described as follows:

9.4.1 Firm's Behaviour

Consider two firms entertaining entry into a market for a commodity, say, breakfast cereals with two niches, sweet cereals, J, and healthy cereals H. Payoff matrix is provided in the following table:

Table 4: Firm's Behaviour

		Firm2	
		Niche J	Niche H
Firm 1	Niche J	(-10,10)	(15,5)
	Niche H	(5,15)	(-5,-5)

As shown in Table-4, if Firm 1 chooses Niche J, Firm 2 will follow Niche H. As Firm 2 chooses Niche H, Firm 1's optimal choice will be Niche J. So Niche J (Firm 1), Niche H (firm 2) will be Nash Equilibrium. On the other hand, if Firm 1 chooses to Niche H, Firm 2 will follow Niche J. As Firm 2 chooses Niche J, Firm 1's optimal choice will be Niche H. so this time, Niche H J (Firm 1), Niche J (firm 2) will be Nash Equilibrium. However, Nash Equilibrium will not be unique.

9.4.2 Firm's Behaviour, Mixed Strategy

Since in the above game, the Nash equilibrium is not unique, the game should be solved by applying mixed strategy.

Table-5: Firm's Behaviour, Mixed Strategy

		Firm 2	
		y Niche J	1-y Niche H
Firm 1	x Niche J	(-10,10)	(15,5)
	1-x Niche H	(5,15)	(-5,-5)

As shown in the payoff matrix, let Firm 1 gives x times to Niche J and (1-x) time to Niche H and Firm 2 gives y times to Niche J and (1-y) time to Niche H.

Firm 1's expected payoff if Firm to plays Niche J: $E_{11}=x(-10)+(1-x)5=5-15x$

Firm 1's expected payoff if Firm to plays Niche H: $E_{12}=x(15)+(1-x)(-5)=20x-5$

Equating Firm 1's expected payoff, i.e,

$$E_{11} = E_{12}$$

$$\text{Or, } 5-15x=20x-5 \Rightarrow x=2/7 \text{ and } (1-x) = 5/7$$

Similarly, Firm 2's expected payoff if Firm to plays Niche J: $E_{21}=y(-10)+(1-y)5=5-15y$

Firm 2's expected payoff if Firm to plays Niche H: $E_{22}=y(15)+(1-y)(-5)=20y-5$

Equating Firm 2's expected payoff, i.e,

$$E_{21} = E_{22}$$

$$\text{Or, } 5-15y=20y-5 \Rightarrow y=2/7 \text{ and } (1-y) = 5/7$$

Thus, Firm 1 gives 2/7 times to Niche J and 5/7 time to Niche H and Firm 2 gives 2/7 times to Niche J and 5/7 time to Niche H. the value of game for Firm 1 and 2 will be 5/7.

9.4.3 Example of Dominant Strategy

Table-5: Firm's Behaviour, Dominant Strategy

		Firm 2	
		High price	Low price
Firm 1	High price	(-10,10)	(3,5)
	Low price	(5,15)	(5,-5)

In the above game, for Firm 1, Low Price strategy will dominate High Price strategy. Therefore, Firm 1 will delete the strategy High Price. The game will be like above.

Table-5.1: Firm's Behaviour, Dominant Strategy

		Firm 2	
		High price	Low price
Firm 1	High price		
	Low price	(5,15)	(5,-5)

Similarly, for Firm 2, High Price strategy will dominate Low Price strategy. Therefore, Firm 2 will delete the strategy Low Price. The game will be like follows. Thus optimum strategy for firm 1 will be Low Price while for Firm it will be High Price.

10. REAL-WORLD AND GAME THEORY

Once we recognize the general idea, we will see such dilemmas everywhere. Competing stores who undercut each other's prices when both would have done better if both had kept their prices high are victims of the dilemma. (But in this instance, consumers benefit from the lower prices when the sellers fink on each other.) The same concept explains why it is difficult to raise voluntary contributions, or to get people to volunteer enough time, for worthwhile public causes.

How might such dilemmas be resolved? If the relationship of the players is repeated over a long time horizon, then the prospect of future cooperation may keep them from finking; this is the well-known tit-for-tat strategy. A "large" player who suffers disproportionately more from complete finking may act cooperatively even when the small fry are finking. Thus Saudi Arabia acts as a swing producer in OPEC, cutting its output to keep prices high when others produce more; and the United States bears a disproportionate share of the costs of its military alliances. Finally, if the group as a whole will do better in its external relations if it enjoys internal cooperation, then the process of biological or social selection may generate instincts or social norms that support cooperation and punish cheating. The innate sense of fairness and justice that is observed among human subjects in many laboratory experiments on game theory may have such an origin.

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